Elgin Academy
Marking Scheme - Advanced Higher Grade 2006/2007
Prelim (Assessing Units 1 & 2)

	Give one mark for each •	**Illustrations for awarding each mark**
1(*a*)	**ans:** $f'(x) = 2e^{-2x}(2\sec^2 4x - \tan 4x)$ **3 marks** • knows to use product rule • differentiates e^{-2x} correctly • differentiates $\tan 4x$	• • $-2e^{-2x}$ • $4\sec^2 4x$
1(*b*)	**ans:** $\frac{dy}{dx} = \frac{x(1-\ln 5x)-1}{x(x-1)^2}$ **3 marks** • knows to use the quotient rule • differentiates correctly • correct simplification for $\frac{dy}{dx}$	• • $\frac{\frac{x-1}{x} - \ln 5x}{(x-1)^2}$ • $\frac{x-1-x\ln 5x}{x(x-1)^2}$
2	**ans:** $t = \frac{31}{9}$ **5 marks** • correct augmented matrix • first modified system correct • second modified system correct • third modified system correct • solves for t	• $\begin{pmatrix} 1 & 2 & -3 & -7 \\ 4 & -1 & 2 & 9 \\ 3 & -2 & t & 13 \end{pmatrix}$ • $\begin{pmatrix} 1 & 2 & -3 & -7 \\ 0 & -9 & 14 & 37 \\ 3 & -2 & t & 13 \end{pmatrix}$ • $\begin{pmatrix} 1 & 2 & -3 & -7 \\ 0 & -9 & 14 & 37 \\ 0 & -8 & t+9 & 34 \end{pmatrix}$ • $\begin{pmatrix} 1 & 2 & -3 & -7 \\ 0 & -9 & 14 & 37 \\ 0 & 0 & t-\frac{31}{9} & \frac{10}{9} \end{pmatrix}$ • $t=\frac{31}{9}$

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3	**ans:** Proof, $(z-3)(z+1)(z^2-2z+10)$ **5 marks** • verifies that $1-3i$ is a solution • knows that $1+3i$ is a solution • uses $1+3i$ for substitution or synthetic division • finds $z^2-2z-3=(z-3)(z+1)$ • finds $z^2-2z+10$ factor	• correct substitution or synthetic division • $1+3i$ is a solution • correct substitution or synthetic division • $z^2-2z-3=(z-3)(z+1)$ • $z^2-2z+10$
4	**ans:** Proof **6 marks** • starts substitution • changes limits correctly • correct substitution • deals with denominator • correctly integrates • substitutes limits correctly	• $dx=-3\sin\theta\ d\theta$ • $\frac{3}{2}\to\frac{\pi}{3},3\to 0$ • $\int_{\frac{\pi}{3}}^{0}\frac{-3\sin\theta d\theta}{\sqrt{9-9\cos^2\theta}}$ • $\int_{\frac{\pi}{3}}^{0}\frac{-3\sin\theta d\theta}{3\sin\theta}$ • $-\left[\theta\right]_{\frac{\pi}{3}}^{0}$ • $\frac{\pi}{3}$
5	**ans:** $243a^{10}-\frac{1620a^8}{b}+\frac{4320a^6}{b^2}-\frac{5760a^4}{b^3}+\frac{3840a^2}{b^4}-\frac{1024}{b^6}$ **3 marks** • correct binomial expression • correct expansion • correct simplification	• $\sum_{r=0}^{5}\binom{5}{r}\left(3a^2\right)^{5-r}\left(\frac{-4}{b}\right)^r$ • $\left(3a^2\right)^5+5\left(3a^2\right)^4\left(\frac{-4}{b}\right)+10\left(3a^2\right)^3\left(\frac{-4}{b}\right)^2$ $10\left(3a^2\right)^2\left(\frac{-4}{b}\right)^3+5\left(3a^2\right)\left(\frac{-4}{b}\right)^4+\left(\frac{-4}{b}\right)^5$ • answer

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6	**ans:** $2 - 5e^{-1}$ **5 marks** • uses integration by parts correctly • uses integration by parts for a second time • integrates correctly • substitutes limits correctly • correct evaluation	• $\left[-x^2e^{-x}\right]_0^1 + \int_0^1 2xe^{-x}dx$ • $\left[-2xe^{-x}\right]_0^1 + \int_0^1 2e^{-x}dx$ • $\left[-2e^{-x}\right]_0^1$ • $-e^{-1} - 2e^{-1} - 2e^{-1} + 2e^{0}$ • $2 - \frac{5}{e}$
7	**ans:** Neither **3 marks** • knows to find $f(-x)$ • finds $f(-x)$ correctly • correct conclusion	• $f(-x) = (-x)^2\cos(-x) + (-x)^3$ • $f(-x) = x^2\cos x - x^3$ • Neither
8	**ans:** 0·35 [cm/s] **5 marks** • knows how find $\frac{dr}{dt}$ • finds $\frac{dr}{dV}$ correctly • finds correct formula for $\frac{dr}{dt}$ • finds correct radius • evaluates $\frac{dr}{dt}$ correctly	• $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ • $\frac{dr}{dV} = \frac{1}{4\pi r^2}$ • $\frac{dr}{dt} = \frac{15}{14r^2}$ • $r = 1{\cdot}75$ • $\frac{dr}{dt} = 0{\cdot}35$
9	**ans:** Proof **3 marks** • knows how to start proof : $n = 2k \pm 1$ • continues proof : simplifies $n = 2k \pm 1$ • completes proof : common factor of 8	• n is odd $\Rightarrow n = 2k \pm 1$ $(k \in Z)$ • $\Rightarrow n^4 - 1 = 16k^4 \pm 32k^3 + 24k^2 \pm 8k$ • $\Rightarrow n^4 - 1 = 8(2k^4 \pm 4k^3 + 3k^2 \pm k)$ which is divisible by 8

	Give one mark for each •	Illustrations for awarding each mark
10(*a*)	**ans:** $\frac{3}{2(x-3)} - \frac{3}{2(x+3)}$ **2 marks** • first fraction • second fraction	 • $\frac{3}{2(x-3)}$ • $-\frac{3}{2(x+3)}$
10(*b*)	**ans:** $1+\frac{3}{2}\ln\frac{1}{2}$ **4 marks** • divides correctly • integrates correctly • substitutes limits correctly • evaluates correctly	 • $1+\frac{9}{x^2-9}$ • $x+\frac{3}{2}\ln\lvert x-3\rvert-\frac{3}{2}\ln\lvert x+3\rvert$ • $\left(1+\frac{3}{2}\ln\lvert-2\rvert-\frac{3}{2}\ln\lvert 4\rvert\right)-\left(0+\frac{3}{2}\ln\lvert-3\rvert-\frac{3}{2}\ln\lvert 3\rvert\right)$ • $1+\frac{3}{2}(\ln 2-\ln 4)$
11(*a*)	**ans:** $x = -3$ & $y = x - 3$ **3 marks** • states equation of vertical asymptote • divides correctly • states equation of oblique asymptote	 • $x = -3$ • $f(x) = x-3+\frac{12}{x+3}$ • $y = x - 3$
11(*b*)	**ans:** (0,0) $\rightarrow$ minimum turning point; (-6,-12) $\rightarrow$ maximum turning point **5 marks** • differentiates correctly • finds *x*-coordinates of stationary points • finds *y*-coordinates of stationary points • finds second derivative or nature table • correct nature of both points	 • $f'(x) = \frac{x^2+6x}{(x+3)^3}$ • $f'(x) = 0 \Rightarrow x = 0,-6$ • (0,0) & (-6,-12) • $f''(x) = \frac{(2x+6)(x+3)^2 - 2(x^2+6x)(x+3)}{(x+3)^4}$ • $f''(0)>0 \Rightarrow (0,0)\,Min.T.P.$ & $f''(-6)<0 \Rightarrow (-6,-12)\,Max.T.P.$

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11(*c*)	**ans:** correct graph **2 marks** • turning points shown • completes graph ***see graph on next page***	 • correct turning points • correct behaviour at asymptotes
11(*d*)	**ans:** (0,10) & (-6,22) **2 marks** • one point correct • second point correct	 • (0,10) • (-6,22)
12(*a*)	**ans:** Proof **4 marks** • knows how to find common difference • simplifies correctly • knows how to find sum of first 10 terms • simplifies correctly	 • $1+\frac{1}{\sqrt{2}}-\left(1+\sqrt{2}\right)$ • $\frac{1}{\sqrt{2}}-\sqrt{2}=\frac{1-2}{\sqrt{2}}=\frac{-1}{\sqrt{2}}$ • $\frac{10}{2}\left[2(1+\sqrt{2})+(10-1)\left(\frac{-1}{\sqrt{2}}\right)\right]$ • $5\left(2+2\sqrt{2}-\frac{9}{\sqrt{2}}\right)=\ldots=\frac{5}{2}(4-5\sqrt{2})$
12(*b*)	**ans:** Proof **5 marks** • knows to find common ratio • finds common ratio correctly • justifies that sum to infinity exists • knows how to find sum to infinity • simplifies correctly	 • $\frac{1+\frac{1}{\sqrt{2}}}{1+\sqrt{2}}$ • $\frac{1}{\sqrt{2}}$ • $-1<\frac{1}{\sqrt{2}}<1$ • $\frac{1+\sqrt{2}}{1-\frac{1}{\sqrt{2}}}$ • $\frac{\sqrt{2}+2}{\sqrt{2}-1}=\ldots=4+3\sqrt{2}$

	Give one mark for each •	Illustrations for awarding each mark
13	**ans:** $t = 3$ **6 marks** • knows how to find volume of solid • finds limits of integration • integrates correctly • substitutes limits correctly • equates volumes • solves for t correctly	• $V = \int \pi(y-4)dy$ • $1 \to 5, t \to t^2 + 4$ • $\pi\left[\frac{y^2}{2} - 4y\right]$ • $\pi\left\{\left(\frac{(t^2+4)^2}{2} - 4(t^2+4)\right) - \left(\frac{5^2}{2} - 4(5)\right)\right\}$ • $40\pi = \pi\left(\frac{t^4}{2} - \frac{1}{2}\right)$ • $t = 3$

TOTAL MARKS = 74

Q11 (c)