

ELGIN ACADEMY

Prelim Examination 2007 / 2008

MATHEMATICS

Advanced Higher Grade

Time allowed - 2 hours

Read Carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**
4. **This examination paper contains questions graded at all levels.**

All questions should be attempted

1. (a) Given $f(x) = 6 \tan^{-1} \sqrt{x}$, where $x > 0$, obtain $f'(x)$ and simplify your answer. **3**

(b) Given $y = x^{x-2}$, where $x > 2$, use logarithmic differentiation to obtain $\frac{dy}{dx}$ in terms of x . **3**

2. $z_1 = 2i$ and $z_2 = 1 - i$.

(a) Express $\frac{z_1}{z_2}$ in the form $a + bi$ (where a and b are real numbers). **2**

(b) Find $\arg\left(\frac{z_1}{z_2}\right)$. **1**

3. Find the term independent of p in the expansion of $\left(3p^3 - \frac{2}{p}\right)^4$. **4**

4. Prove by induction that for all natural numbers n , $2^{3n} - 1$ is divisible by 7. **5**

5. (a) Show that the matrix $A = \begin{pmatrix} 2 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 3 & 1 \end{pmatrix}$ is non-singular. **3**

(b) Use elementary row operations to find A^{-1} . **5**

6. Use integration by parts to evaluate

$$\int_0^1 2 \tan^{-1} x \, dx. \quad \mathbf{4}$$

7. A curve is defined by the parametric equations

$$x = t^2 - 2t, \quad y = 1 - t^4.$$

Find the equation of the tangent to the curve at the point where $t = -1$. 4

8. Express the improper rational function $f(x) = \frac{x^3 + 3x^2 - 8x + 2}{x^2 - 2x + 1}$ in the form

$$f(x) = g(x) + h(x),$$

where $g(x)$ is a polynomial function and $h(x)$ is a proper rational function expressed in partial fractions. 6

9. By using the substitution $t = 1 + \tan x$, show that:

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1 + \tan x} dx \quad 5$$

10. (a) Calculate the sum of all the two digit natural numbers which are divisible by 3. 4

- (b) Find the value of θ , $0 < \theta < \frac{\pi}{2}$, such that:

$$1 + \sin^2 \theta + \sin^4 \theta + \sin^6 \theta + \dots = 2. \quad 5$$

11. A scientist constructs the differential equation

$$\frac{dy}{dx} = e^{x+y}$$

to describe the relationship between two quantities x and y .

- (a) Find the general solution of the differential equation. 4

- (b) Given that $y = 0$ when $x = 1$, find the particular solution, expressing y in terms of x . 2

12. The function f is defined by $f(x) = \frac{x^2 + 3}{x + 1}, x \neq -1, x \in R$.

- (a) (i) Write down the equation of the vertical asymptote of f . **1**
- (ii) Show that f has a non-vertical asymptote and obtain its equation. **2**
- (iii) Find the point(s) of intersection with the x - and y - axes. **2**
- (b) Find the coordinates and nature of the stationary points of f . **5**
- (c) Sketch the graph of $y = f(x)$, indicating the features found in (a) and (b). **3**

[END OF QUESTION PAPER]