Marking Scheme – Advanced Higher Grade 2008 / 2009
Prelim (Assessing Units 1 & 2)

	Give one mark for each •	Illustrations for awarding each mark
1.	**ans:** $\frac{dy}{dx}=\frac{\sqrt{1-t^2}}{t}$ **3 marks** • knows how to use parametric differentiation • differentiates correctly • simplifies correctly	• $\frac{y'(t)}{x'(t)}$ • $\frac{\frac{1}{t}}{\frac{1}{\sqrt{1-t^2}}}$ • $\frac{\sqrt{1-t^2}}{t}$
2.	**ans:** 1120 **5 marks** • finds correct general term • simplifies to find correct expression for power of x • solves for r correctly • substitutes correctly • finds correct coefficient of x^{-4}	• $\binom{8}{r}\left(\frac{x^2}{2}\right)^{8-r}\left(-\frac{4}{x^3}\right)^r$ • x^{16-5r} • $r=4$ • $\binom{8}{4}\frac{(-4)^4}{2^4}x^{-4}$ • 1120
3.	**ans:** $\frac{6}{x^2+9}+\frac{2}{x+3}$ **4 marks** • know how to find partial fractions • know how to find A, B and C • finds A • finds B and C	• $\frac{Ax+B}{x^2+9}+\frac{C}{x+3}$ • $2x^2+6x+36=(x+3)(Ax+B)+C(x^2+9)$ • $A=0$ • $B=6$ and $C=2$
4.	**ans:** $z=2+i$ $z=2-i, z=i$ & $z=-i$ **6 marks** • starts correctly • simplifies correctly • verifies correctly • correct conjugate root • correct equation • correct roots	• $\frac{5i}{1+2i}\frac{1-2i}{1-2i}$ • $2+i$ • '$R=0$' • 2 - i • $z^2+1=0$ • $\pm i$

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5.	**ans:** Proof **5 marks** • verifies result for $n = 1$ (e.g.) • states correct assumption for $n = k$ • states correct result for $n = k + 1$ • continues proof correctly • concludes proof correctly	• $8^1 - 7(1) + 6 = 7$ • $8^k - 7k + 6 = 7m$ • $8^{k+1} - 7(k+1) + 6 = 7l$ • $8^{k+1} - 7(k+1) + 6 = ... = 7(8m + 7k - 7)$ • $\therefore$ Since T for $n = 1$ & (T for $n = k \Rightarrow$ T for $n = k+1$), the result is T $\forall n \in N$
6.	**ans:** $\frac{dy}{dx} = \frac{2y - 3x^2y^2}{2x^3y - 2x}$ $3y = -8x - 14$ **6 marks** • uses implicit differentiation correctly • rearranges correctly • correct answer • correct y value • correct gradient • correct equation	• $3x^2y^2 + 2x^3y\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} = 0$ • $(2x^3y - 2x)\frac{dy}{dx} = 2y - 3x^2y^2$ • $\frac{dy}{dx} = \frac{2y - 3x^2y^2}{2x^3y - 2x}$ • $y = -2$ • $\frac{-8}{3}$ • $y + 2 = \frac{-8}{3}(x + 1)$
7(a)	**ans:** 37 **2 marks** • correct expansion • correct answer	• $16 \times \left(\frac{3}{4}\right)^0 + 16 \times \left(\frac{3}{4}\right)^1 + 16 \times \left(\frac{3}{4}\right)^2$ • 37
7(b)	**ans:** $-1 < \frac{3}{4} < 1$ **3 marks** 64 • correct explanation • uses correct formula • correct answer	• $-1 < \frac{3}{4} < 1$ • $S_n = \frac{16}{1 - \frac{3}{4}}$ • 64

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8.	**ans:** No solutions **5 marks** • correct augmented matrix • first modified system correct • second modified system correct • third modified system correct • correct conclusion	• $\begin{bmatrix} 2 & -7 & 10 & -1 \\ 1 & -3 & 4 & 2 \\ 5 & -18 & 26 & -6 \end{bmatrix}$ • $\begin{bmatrix} 1 & -3 & 4 & 2 \\ 0 & -1 & 2 & -5 \\ 5 & -18 & 26 & -6 \end{bmatrix}$ • $\begin{bmatrix} 1 & -3 & 4 & 2 \\ 0 & -1 & 2 & -5 \\ 0 & -3 & 6 & -16 \end{bmatrix}$ • $\begin{bmatrix} 1 & -3 & 4 & 2 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
9(*a*)	**ans:** $1+\dfrac{9}{x-1}$ **2 marks** • knows to use algebraic long division • divides correctly	• • $1+\dfrac{9}{x-1}$
9(*b*)	**ans:** Proof **6 marks** • knows to integrate • correct integral • correct limits • integrates correctly • substitutes correctly • completes proof	• $A=\int\ldots$ • $\int\left(12-x-1-\dfrac{9}{x-1}\right)dx$ • $\int_2^{10}\ldots$ • $\left[11x-\dfrac{x^2}{2}-9\ln\lvert x-1\rvert\right]$ • $(110-50-9\ln 9)-(22-2-9\ln 1)$ • $40-\ln 9^9$.

	Give one mark for each •	Illustrations for awarding each mark
10(*a*)	**ans:** $I=\int_1^3 \frac{1}{2}ue^u\,du$ **3 marks** • starts substitution correctly • continues substitution correctly • changes limits correctly	• $\frac{du}{dx}=2(4x+1)^{-\frac{1}{2}}$ • $\frac{u}{2}du=dx$ • $0\to 1\ \&\ 2\to 3$
10(*b*)	**ans:** e^3 **5 marks** • uses integration by parts correctly • evaluates correctly • integrates correctly • evaluates correctly • correct answer	• $\left[\frac{u}{2}e^u\right]-\int\frac{1}{2}e^u\,du$ • $\frac{3}{2}e^3-\frac{1}{2}e$ • $\ldots-\left[\frac{1}{2}e^u\right]$ • $\ldots-\frac{1}{2}e^3+\frac{1}{2}e$ • e^3
11(*a*)	**ans:** Neither **2 marks** • correct formula for $g(-x)$ • correct conclusion	• $g(-x)=e^{-2x}\sin(-2x)$ • Neither
11(*b*)	**ans:** $\left(\frac{3\pi}{8},\frac{e^{\frac{3\pi}{4}}}{\sqrt{2}}\right)$ **5 marks** • differentiates correctly • sets derivative equal to zero • solves correctly • solves correctly • correct y-value	• $g'(x)=2e^{2x}\sin 2x+2e^{2x}\cos 2x$ • $2e^{2x}(\sin 2x+\cos 2x)=0$ • $e^{2x}\neq 0$ • $x=\frac{3\pi}{8}$ • $y=\frac{e^{\frac{3\pi}{4}}}{\sqrt{2}}$
11(*c*)	**ans:** $g''(x)=8e^{2x}\cos 2x$ **1 mark** • correct answer	• $g''(x)=8e^{2x}\cos 2x$
11(*d*)	**ans:** Maximum turning point **2 marks** • correct substitution • correct conclusion	• $g''\left(\frac{3\pi}{8}\right)=\frac{8e^{\frac{3\pi}{4}}}{-\sqrt{2}}$ • Maximum turning point

	Give one mark for each •	Illustrations for awarding each mark
12(*a*)	**ans:** $z^3 = \cos 3\theta + i\sin 3\theta$ **1 mark** • correct answer	• $z^3 = \cos 3\theta + i\sin 3\theta$
12(*b*)	**ans:** $z^3 =$ $\cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$ **2 marks** • expands correctly • simplifies correctly	• $(\cos\theta)^3 + 3(\cos\theta)^2(i\sin\theta) +$ $3(\cos\theta)(i\sin\theta)^2 + (i\sin\theta)^3$ • $\cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$
12(*c*) (i)	**ans:** $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ **2 marks** • equates real parts correctly • correct expression	• $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$ • $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
12(*c*) (ii)	**ans:** $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ **2 marks** • equates imaginary parts correctly • correct expression	• $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$ • $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
12(*d*)	**ans:** Proof **4 marks** • starts correctly • simplifies correctly • substitutes correctly • completes proof	• $\dfrac{\cos 3\theta}{\sin 3\theta}$ • $\dfrac{4 - 3\sec^2\theta}{3\tan\theta\sec^2\theta - 4\tan^3\theta}$ • $\sec^2\theta = 1 + \tan^2\theta$ • $\cot 3\theta = \dfrac{1 - 3\tan^2\theta}{3\tan\theta - \tan^3\theta}$

TOTAL MARKS = 76