

ELGIN ACADEMY

Prelim Examination 2006 / 2007

(Assessing Units 1 & 2 + Wave Function + Expo. & Logs)

MATHEMATICS Higher Grade - Paper II

Time allowed - 1 hour 30 minutes

Read Carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

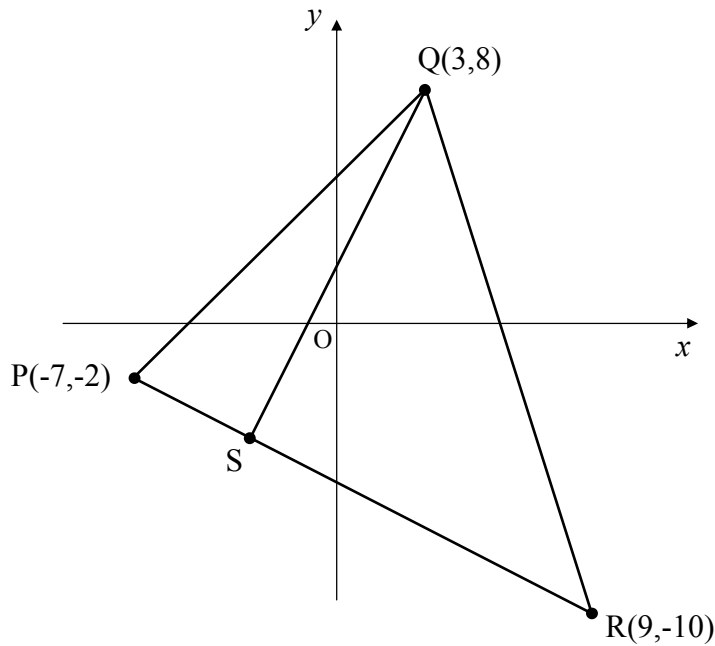
$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

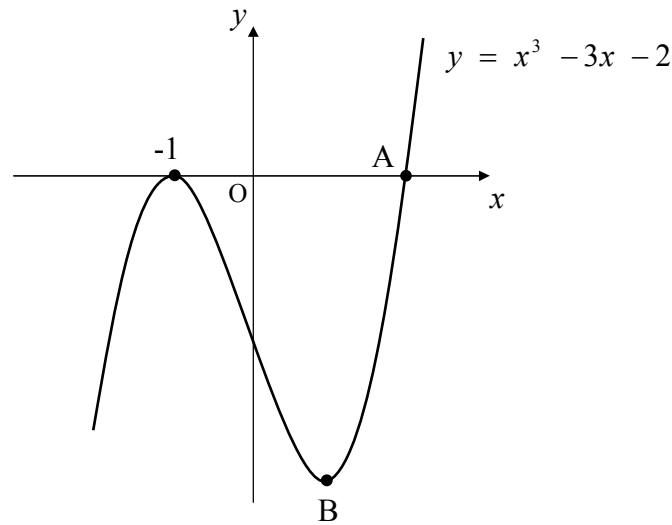
All questions should be attempted

1. Triangle PQR has as its vertices $P(-7,-2)$, $Q(3,8)$ and $R(9,-10)$ as shown.



- (a) Find the equation of side PR. 2
- (b) Find the equation of the **altitude** QS. 3
- (c) Hence find the coordinates of S, the point where the altitude QS meets side PR. 4
- (d) Establish the equation of the circle which passes through the points Q, S and R. 4
2. A recurrence relation is defined as $u_{n+1} = 0.75u_n + 12$.
- Given that $U_0 = 32$, find the **difference** between the limit of the sequence and the third term, U_3 . 5
3. A curve has as its equation $y = (x - 6)^2 + 8$.
- Given that the line with equation $y = 2x - 5$ is a tangent to this curve, establish the co-ordinates of the point T, the point of contact between the curve and the line. 4

4. Part of the graph of the curve with equation $y = x^3 - 3x - 2$ is shown below. The curve passes through the point $(-1, 0)$.

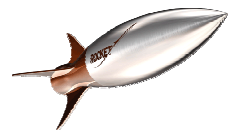


Find the coordinates of the root A, and the stationary point B.

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5. A circle has as its equation $(x - 9)^2 + (y + 1)^2 = 117$.
- (a) Given that the point $A(3, k)$ lies on this circle, find k where $k > 0$. 4
- (b) Find the equation of the tangent to this circle at the point A. 4
- (c) Show clearly that this tangent passes through the centre of the circle with equation $x^2 + y^2 + 6x - 8y + 12 = 0$. 2

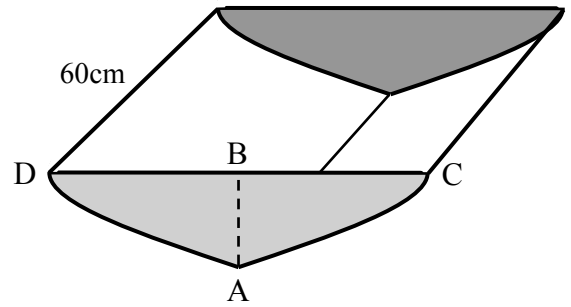
6. A rocket's supply of fuel decays according to the formula $M_t = M_0 e^{-0.04t}$, where M_0 is the initial supply in tonnes and M_t is the fuel remaining after t seconds.



Given an initial fuel supply of 60 tonnes, calculate, to the nearest second, how long the rocket takes to **use up** 36 tonnes of its fuel.

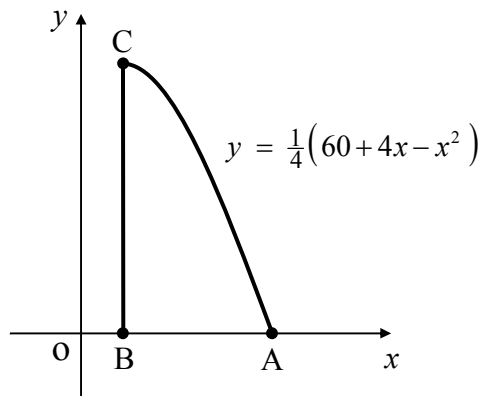
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7. A small feeding trough is shown opposite.
 The end face has an axis of symmetry AB.
 Edge CD is perpendicular to the axis of symmetry.

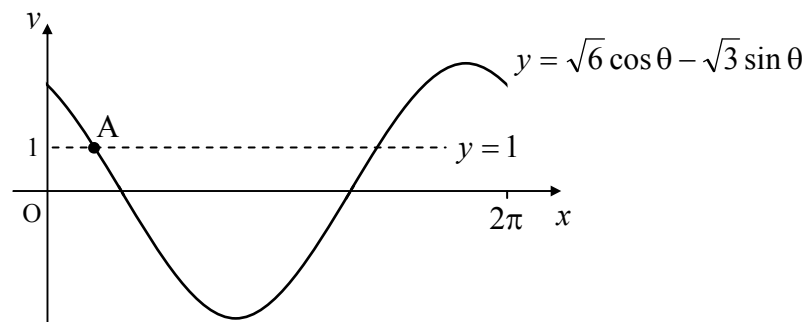


When the end face is rotated through 90° and then halved along the axis of symmetry, shape ABC can be placed on a coordinate diagram as shown below.

AB lies along the x -axis with the curved edge CA being part of the curve with equation $y = \frac{1}{4}(60 + 4x - x^2)$ and BC is a line of symmetry of the curve.



- (a) Establish the coordinates of A and B. 4
- (b) Hence calculate the **area** of shape ABC given that all the units are in centimetres. 4
- (c) Given that the trough is a prism and measures 60cm from back to front, calculate the volume of feed the trough can hold when full, giving your answer correct to the nearest litre. 3
8. The diagram below shows the graph of $y = \sqrt{6} \cos \theta - \sqrt{3} \sin \theta$ for $0 \leq \theta \leq 2\pi$. The line with equation $y = 1$ first intersects this curve at the point A as shown.



Find the x -coordinate of the point A, in radians, correct to 2 decimal places.

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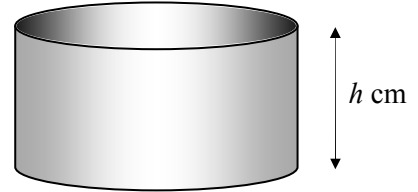
9. An cylindrical container, open at the top, has a volume of 64π cubic centimetres and a height of h centimetres.

(a) Show that $h = \frac{64}{r^2}$.

- (b) If the radius of the container is r cm, show that the total surface area, A , of the container, can be represented by the function

$$A(r) = \frac{128\pi}{r} + \pi r^2.$$

- (c) Hence find the dimensions of the cylinder so that this surface area is a minimum.



1

2

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[END OF QUESTION PAPER]