

ELGIN ACADEMY

Prelim Examination 2006 / 2007

(Assessing Units 1 & 2 + Wave Function + Expo. & Logs)

MATHEMATICS

Higher Grade - Paper I (Non-calculator)

Time allowed - 1 hour 10 minutes

Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A\end{aligned}$$

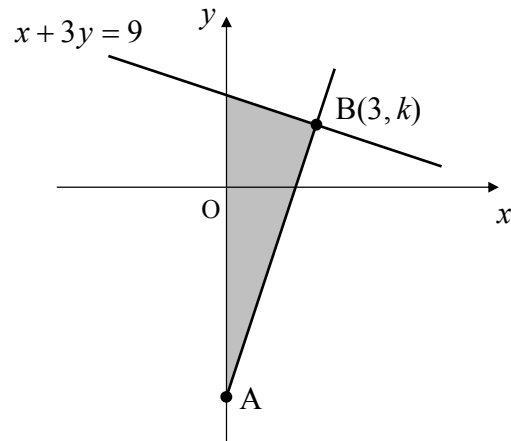
All questions should be attempted

1. A curve has as its derivative $\frac{dy}{dx} = 8x - 3$.

Given that the point $(1, -3)$ lies on this curve, express y in terms of x .

4

2. Part of the line with equation $x + 3y = 9$ is shown in the diagram. B lies on this line and has coordinates $(3, k)$.



- (a) Find the value of k . 1
- (b) Given that the line AB is perpendicular to the line $x + 3y = 9$, find the equation of the line AB. 3
- (c) Hence write down the coordinates of A. 1
- (d) Calculate the area of the shaded triangle. 4

3. A curve has as its equation $y = \frac{x^2 - 4x}{\sqrt{x}}$, where $x \in \mathbb{R}$ and $x > 0$.

Find the gradient of the tangent to this curve at the point where $x = 4$.

6

4. Two functions are defined on suitable domains as $f(x) = x + 1$ and $g(x) = x^2 + 6x + 13$. Given that the function h is such that $h(x) = g(f(x))$, express h in the form

$$h(x) = (x + a)^2 + b, \text{ where } a \text{ and } b \text{ are integers,}$$

and hence write down the minimum value of h and the corresponding replacement for x .

6

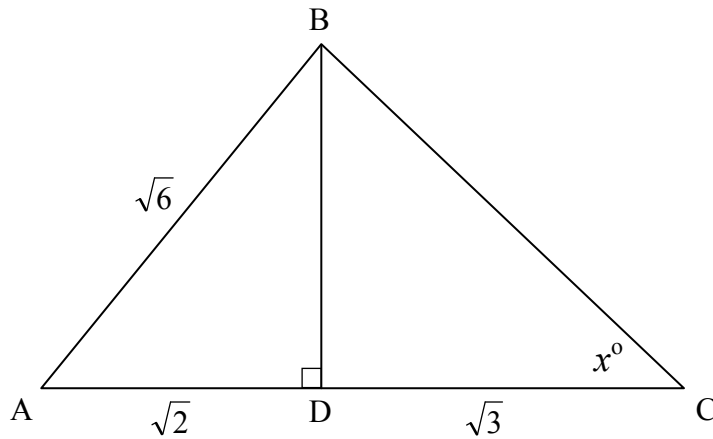
5. Given that $x = -2$ and $x = 1$ are two roots of the equation $x^3 + px^2 - 6x + q = 0$, establish the values of p and q and hence find the third root of the equation.

5

6. For what value of p , where $p > 0$, does the equation $(p^2 + 11)x^2 - 12px + p^2 = 0$ have equal roots?

6

7. In the diagram below $AB = \sqrt{6}$, $AD = \sqrt{2}$ and $DC = \sqrt{3}$. Angle $BCD = x^\circ$.



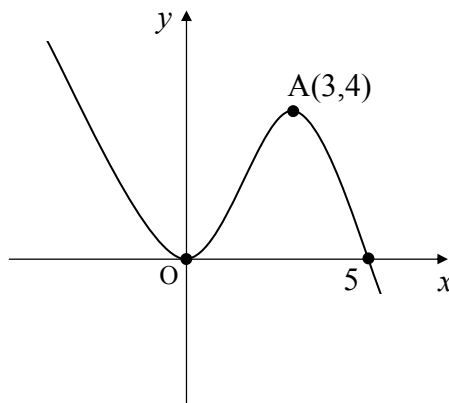
(a) Show clearly that $\tan x^\circ = \frac{2}{\sqrt{3}}$.

2

(b) Hence show that $\sin 2x^\circ = \frac{4}{7}\sqrt{3}$.

3

8. The diagram shows part of the graph of $y = f(x)$.

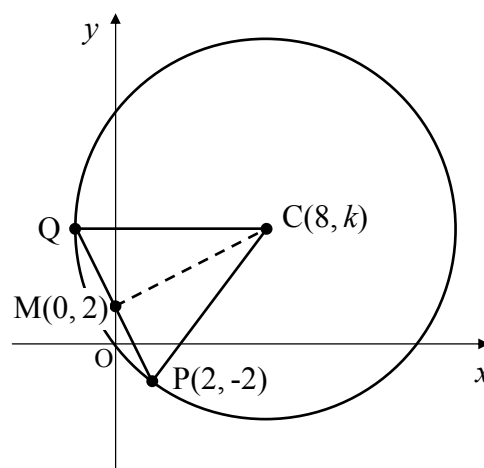


Sketch the graph of $y = -[f(x + 3)]$ marking clearly the **new** positions of the highlighted points and stating their new coordinates.

3

9. A circle, centre $C(8, k)$, has the points $P(2, -2)$ and Q on its circumference as shown.

$M(0, 2)$ is the mid-point of the chord PQ .



- (a) Find the coordinates of Q . 1
- (b) Given that radius CQ is horizontal, write down the value of k , the y -coordinate of C . 1
- (c) Hence establish the equation of the circle. 3

10. Given that $\log_2(x^2 + 8) - 2\log_2 3 = 3$, find the value of x where $x > 0$. 4

11. A sequence of numbers is defined by the recurrence relation $U_{n+1} = kU_n + c$, where k and c are constants.

- (a) Given that $U_2 = 70$, $U_3 = 65$ and $U_4 = 62 \cdot 5$, find **algebraically**, the values of k and c . 3
- (b) Hence find the limit of this sequence. 2
- (c) Express the difference between the fifth term and the limit of this sequence as a percentage of the limit, correct to the nearest percent. 2

[END OF QUESTION PAPER]