

ELGIN ACADEMY

Prelim Examination 2005 / 2006

(Assessing Units 1 , 2 + Exponential/Logs & Wave Function)

MATHEMATICS

Higher Grade - Paper I (Non-calculator)

Time allowed - 1 hour 10 minutes

Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

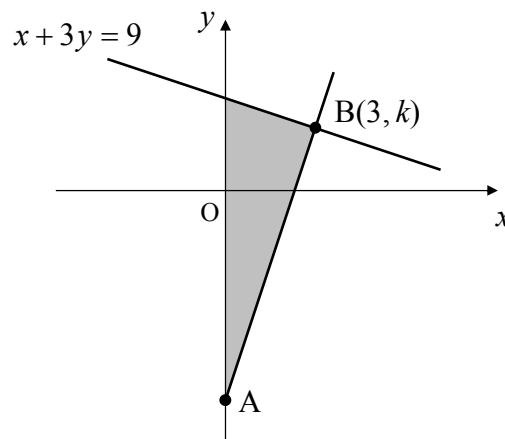
$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

All questions should be attempted

1. Part of the line with equation $x + 3y = 9$ is shown in the diagram. B lies on this line and has coordinates $(3, k)$.



- (a) Find the value of k . 1
- (b) Given that the line AB is perpendicular to the line $x + 3y = 9$, find the equation of the line AB. 3
- (c) Hence write down the coordinates of A. 1

2. (a) A function f has as its derivative $f'(x) = x^3 - ax^2 - 4ax$.

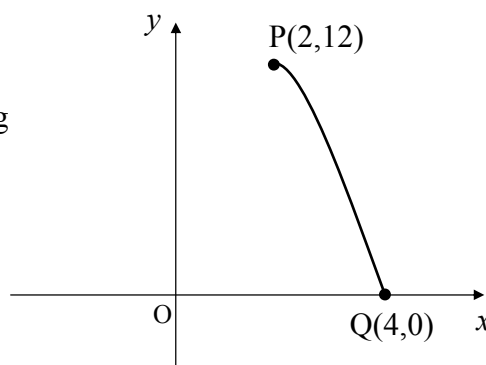
Find a if the function has a stationary point at $x = 4$. 3

- (b) Hence find the rate of change of this function at $x = -2$ and comment on your result. 3

3. A quadratic function, defined on a suitable domain, is given as $f(x) = 12x - 3x^2$.

The diagram shows part of the graph of this quadratic function, $y = f(x)$.

The graph passes through the points P(2,12) and Q(4,0) as shown.



- (a) Sketch the graph of $y = -f(x) + 6$ marking clearly the image points of P and Q and stating their coordinates. 3

- (b) Given that $g(x) = -f(x) + 6$, write down a formula for $g(x)$. 1

4. Find a given that $\int_a^2 (4 + 2x) dx = 0$, where $a < 2$. 4

5. (a) Express $\sin 3x + \cos 3x$ in the form $k \cos(3x - \alpha)$ 4

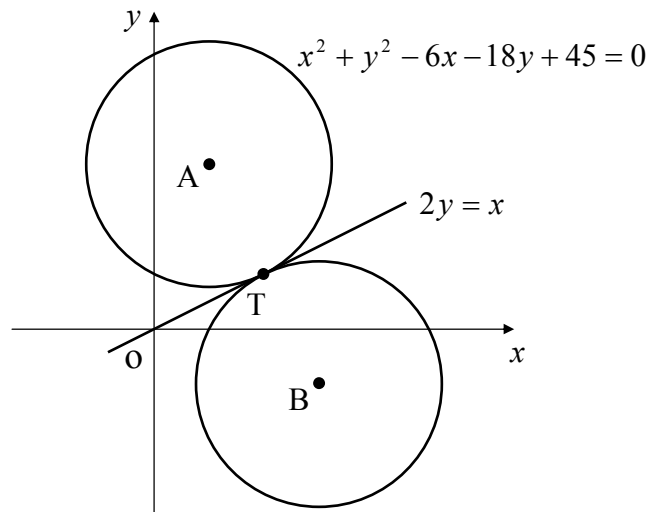
- (b) Solve $\sin 3x + \cos 3x = 1$ for $0 \leq x \leq \frac{\pi}{2}$ 4

6. Two functions are defined on suitable domains as $f(x) = x + 1$ and $g(x) = x^2 + 6x + 13$.
Given that the function h is such that $h(x) = g(f(x))$, express h in the form

$$h(x) = (x + a)^2 + b, \text{ where } a \text{ and } b \text{ are integers,}$$

and hence write down the minimum value of $h(x)$ and the corresponding replacement for x . **6**

7. The diagram below shows two congruent circles which touch at a single point T.
The circle, centre A, has as its equation $x^2 + y^2 - 6x - 18y + 45 = 0$.
The line with equation $2y = x$ is the common tangent to the two circles through T.



- (a) **Show algebraically** that T has coordinates (6,3). **5**
- (b) Hence establish the the coordinates of B, the centre of the lower circle, and find the equation of this circle. **6**
8. The gradient of the tangent to a curve at each point (x, y) is given by $\frac{dy}{dx} = 6x - 6$
If the curve passes through $(1,1)$, find its equation. **4**
9. Find $f'(x)$ when $f(x) = \frac{x^2 - 2\sqrt{x}}{x}$, expressing your answer with positive indices, and hence calculate the value of the gradient of the tangent to the curve $y = f(x)$ at $x = 4$. **6**
10. What can you say about p if the equation, $x + \frac{9}{x} = p$ has **no real** roots? **6**

[END OF QUESTION PAPER]